

Final Exam, MTH 213 , Summer 2022

Ayman Badawi

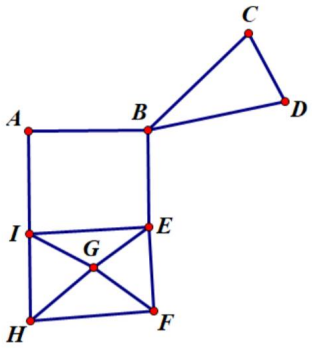
Score = $\frac{\quad}{65}$

QUESTION 1. (i) (6 points) Can we construct a CONNECTED bipartite graph of order 7 with the following sequence of degrees: 4, 3, 1, 1, 1, 1, 1?Show the work. If yes, then draw such connected bipartite graph?

(ii) (3 points) Let G be a connected graph of order 2022 and size 2021. Convince me briefly that such graph is a connected bipartite graph.

(iii) (4 points) Let $G(V, E)$ be a graph with vertex set $V = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $\forall a, b \in V,\ a - -b$ is in E (i.e., $a - -b$ is an edge) if and only $a + b \ \ mod\ (10) \in \{0, 2, 4, 5, 6, 8\}$. By drawing, convince me that G is connected and tell me a reason why it is not a bipartite.

QUESTION 2. Consider the following graph G

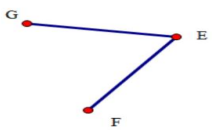


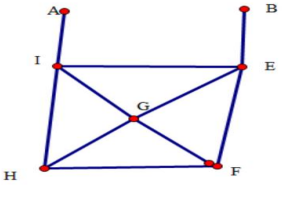
(i) (3 points) Is G an Eulerian circuit? explain. If yes, then construct such circuit.

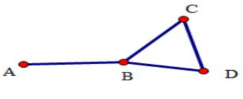
(ii) (3 points) Is G a Hamiltonian ? explain. If yes, then construct such cycle

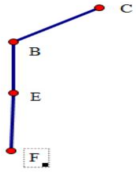
(iii) (3 points) Is G an Euler path? explain. If yes, then construct such path

(iv) (6 points) The following are subgraphs of G. Label each as "subgraph but not induced" or "induced subgraph"









QUESTION 3. (i) (6 points) Given x is a positive integer such that, $210 < x < 420$, $x \ (mod \ 3) = 1$, $x \ (mod \ 7) = 5$, and $x \ (mod \ 10) = 3$. Use the CRT and find the value of x.

(ii) (4 points) Let f be a bijective function such that $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 3 & 1 & 8 & 9 & 6 & 4 & 2 & 7 \end{pmatrix}$. Find the smallest positive integer n such that $f^n = f \ of \ o \cdots o \ f = I$.

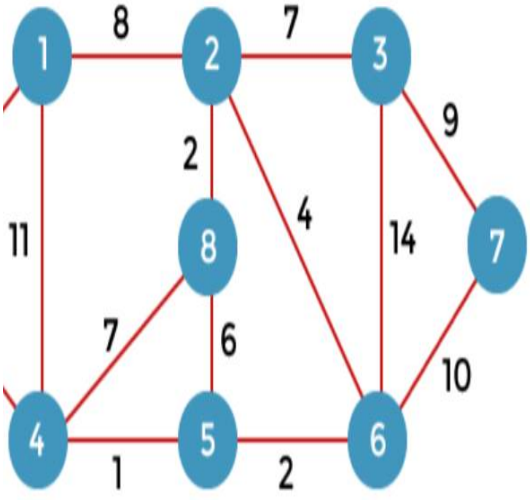
QUESTION 4. (4 points) For every positive integer $n \geq 1$, prove that $10 \mid (3^{4n} - 1)$

QUESTION 5. a. (3 points) In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there?

b. (3 points) In how many different ways can the letters of the word 'JUDGE' be arranged such that "UE" or "EU" always come together?

c. (3 points) From a group of 7 women and 5 men, a committee consisting of 3 men and 3 women is to be formed. In how many ways can the committee be formed if two of the men refuses to serve together?

QUESTION 6. (8 points) Use Dijkstra's Algorithm as explained in the class and CONSTRUCT the minimum weighting spanning tree(i.e., the total weight between every two vertices is minimum)



QUESTION 7. (6 points) Let $a_n = 4a_{n-1} - 3a_{n-2} - 12$ such that $a_1 = 10,\ a_2 = 22$. Find a general formula (equation) for a_n.

Faculty information

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